DEFORMATION OF STRUCTURES
Moment Diagrams by Parts

Basic Principles

1. The bending moment caused by all forces to the left or to the right of ***any section*** is equal to the respective algebraic sum of the bending moments at that section caused by each load acting separately.

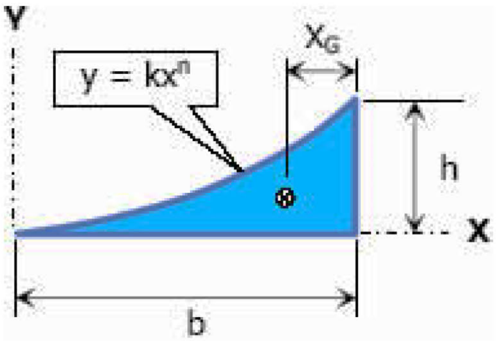
$$M = (\Sigma M)_L = (\Sigma M)_R$$

2. The moment of a load about a specified axis is always defined by the equation of a spandrel.

$$y = k\,x^n$$

where n is the degree of power of x.

The graph of the above equation is as shown below



Area and centroid of moment diagram (spandrel)

and the area and location of centroid are defined as follows.

$$A = \frac{1}{n+1} b h$$

$$X_G = \frac{1}{n+2} b$$

Cantilever Loadings

A = area of moment diagram
M_x = moment about a section of distance x
$\bar{x}$ = location of centoid
Degree = degree power of the moment diagram

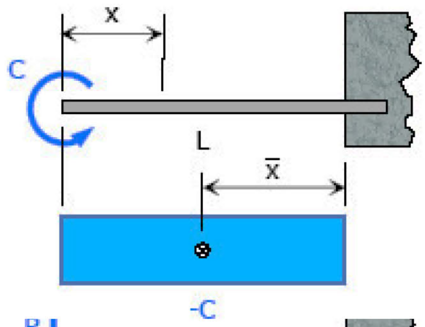
Couple or Moment Load

$$A = -CL$$

$$M_x = -C$$

$$\bar{x} = \frac{1}{2}L$$

Degree : *zero*



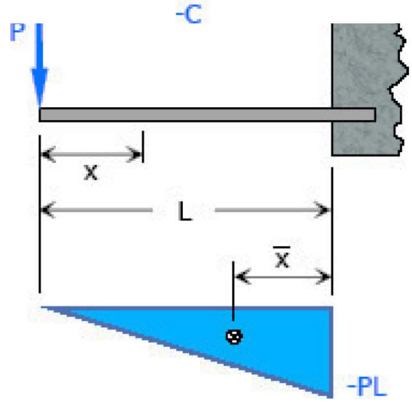
Concentrated Load

$$A = -\frac{1}{2}PL^2$$

$$M_x = -Px$$

$$\bar{x} = \frac{1}{3}L$$

Degree : *first*



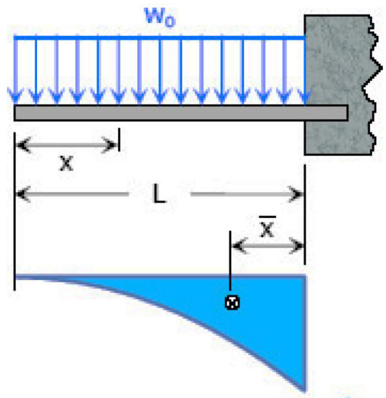
Uniformly Distributed Load

$$A = -\frac{1}{6}w_oL^3$$

$$Mx = -\frac{1}{2}w_ox^2$$

$$\bar{x} = \frac{1}{4}L$$

Degree : *second*



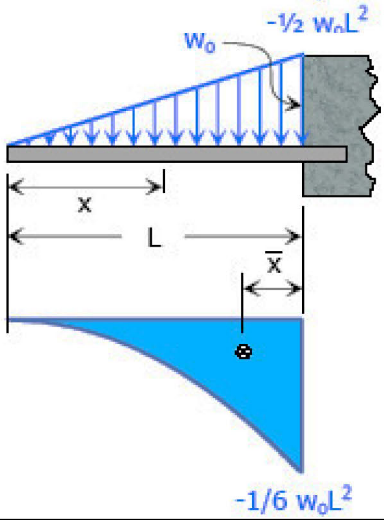
Uniformly Varying Load

$$A = -\frac{1}{24}w_oL^3$$

$$M_x = -\frac{w_o}{6L}x^3$$

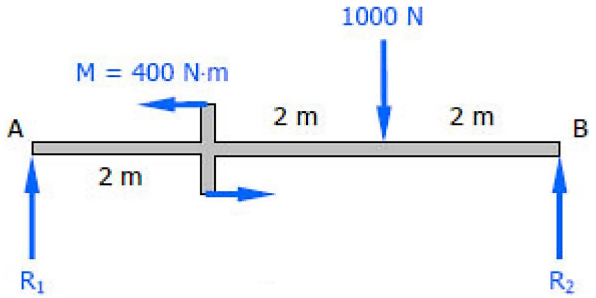
$$x = \frac{1}{5}L$$

Degree : *third*



Example 1

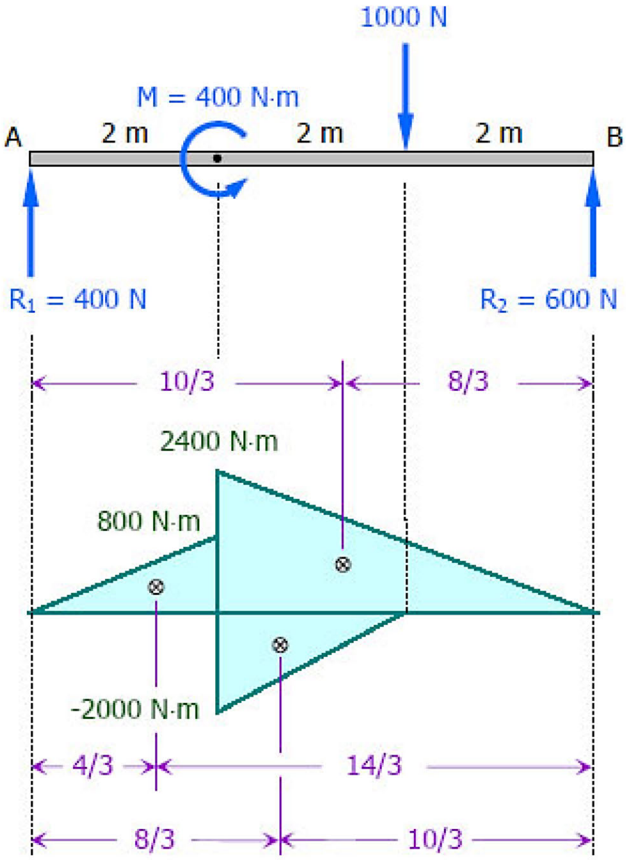
For the beam loaded as shown in the figure , compute the moment of area of the ***M*** diagrams between the reactions about both the left and the right reaction.



Solution

$$\Sigma M_{R2} = 0$$
$$6R_1 = 400 + 1000(2) \quad \Rightarrow \quad R_1 = 400N$$
$$\Sigma M_{R1} = 0$$
$$6R_2 + 400 = 1000(2) \quad \Rightarrow \quad R_2 = 600N$$

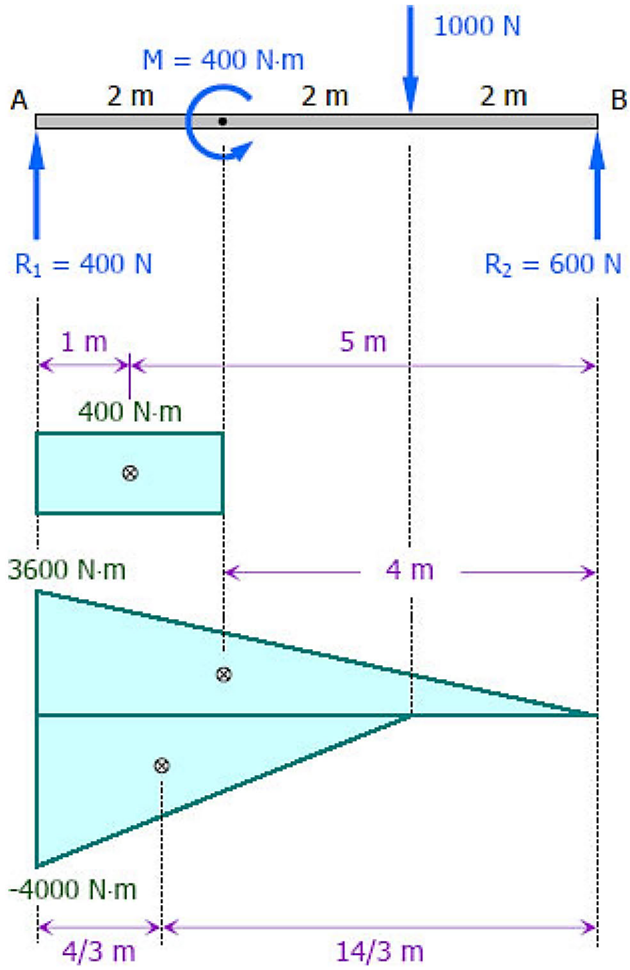
Moment diagram by parts can be drawn in different ways;



<u>1st Solution</u>

$$(Area_{AB})\bar{X}_A = \frac{1}{2}(2)(800)\left(\frac{4}{3}\right) + \frac{1}{2}(4)(2400)\left(\frac{10}{3}\right) - \frac{1}{2}(2)(2000)\left(\frac{8}{3}\right)$$
$$(Area_{AB})\bar{X}_A = 11733.33 \text{ N.m}^3$$
$$(Area_{AB})\bar{X}_B = \frac{1}{2}(2)(800)\left(\frac{14}{3}\right) + \frac{1}{2}(4)(2400)\left(\frac{8}{3}\right) - \frac{1}{2}(2)(2000)\left(\frac{10}{3}\right)$$
$$(Area_{AB})\bar{X}_B = 9866.67 \text{ N.m}^3$$

2nd Solution



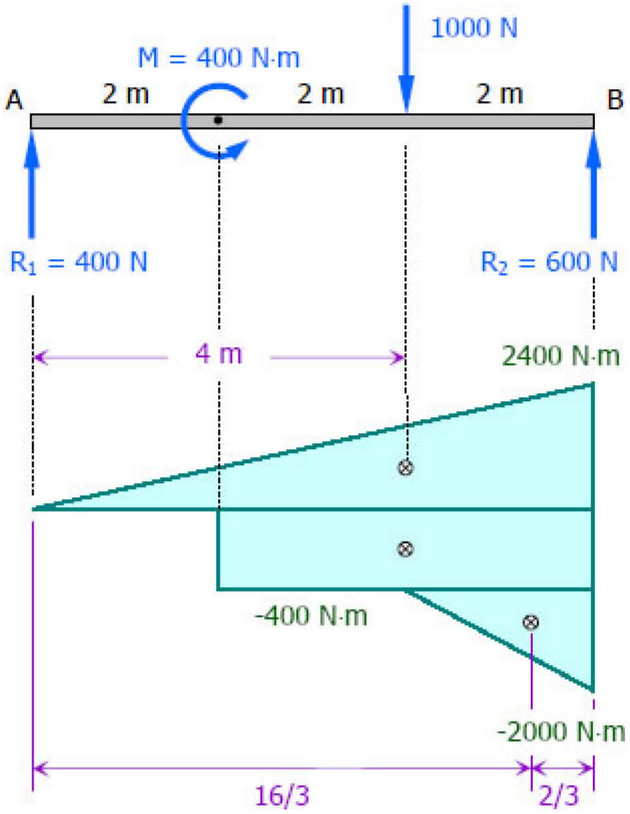
$$(Area_{AB})\bar{X}_A = 400(2)(1) + \frac{1}{2}(6)(3600)(2) - \frac{1}{2}(4)(4000)\left(\frac{4}{3}\right)$$

$$(Area_{AB})\bar{X}_A = 11733.33\ \text{N.m}^3$$

$$(Area_{AB})\bar{X}_B = 400(2)(5) + \frac{1}{2}(6)(3600)(4) - \frac{1}{2}(4)(4000)\left(\frac{14}{3}\right)$$

$$(Area_{AB})\bar{X}_B = 9866.67\ \text{N.m}^3$$

3rd Solution



$$(Area_{AB})\bar{X}_A = \frac{1}{2}(6)(2400)(4) - 400(4)(4) - \frac{1}{2}(2)(2000)\left(\frac{16}{3}\right)$$

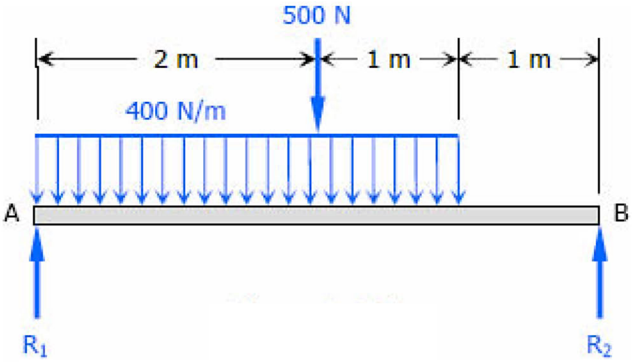
$$(Area_{AB})\bar{X}_A = 11733.33 \ \text{N.m}^3$$

$$(Area_{AB})\bar{X}_B = \frac{1}{2}(6)(2400)(2) - 400(4)(2) - \frac{1}{2}(2)(2000)\left(\frac{2}{3}\right)$$

$$(Area_{AB})\bar{X}_B = 9866.67 \ \text{N.m}^3$$

Example 2

For the beam loaded as shown in the figure, compute the moment of area of the ***M*** diagrams between the reactions about both the left and the right reaction.



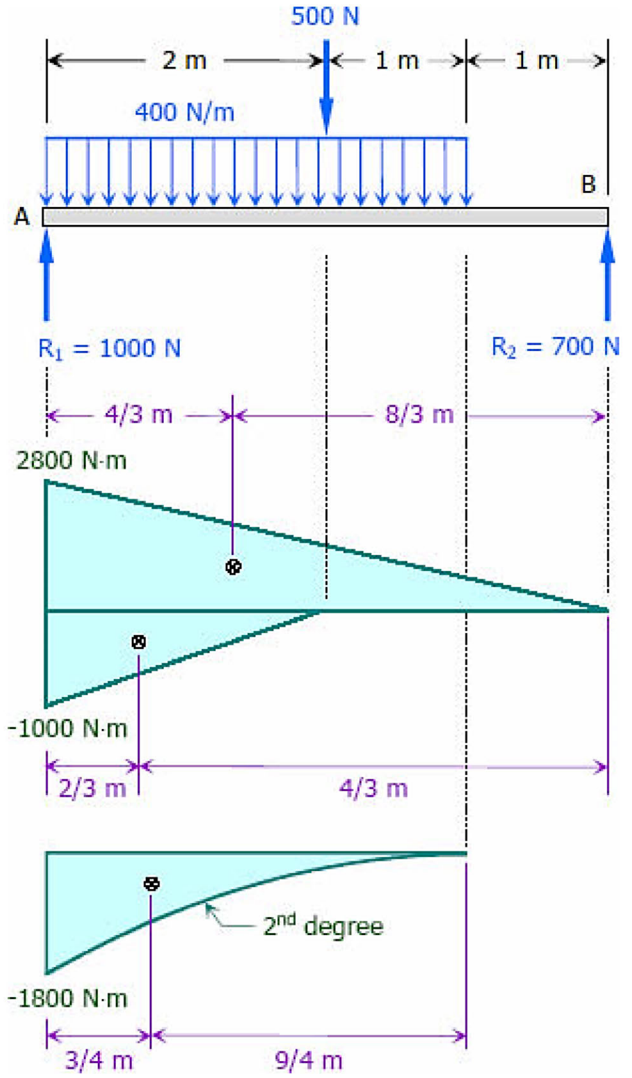
Solution

$$\Sigma M_{R2} = 0$$

$$4R_1 = 400(3)(2.5) + 500(2) \quad \Rightarrow R_1 = 1000\text{N}$$

$$\Sigma M_{R1} = 0$$

$$4R_2 = 400(3)(1.5) + 500(2) \quad \Rightarrow R_2 = 700\text{N}$$



$$(Area_{AB})\bar{X}_A = \frac{1}{2}(4)(2800)\left(\frac{4}{3}\right) - \frac{1}{2}(2)(1000)\left(\frac{2}{3}\right) - \frac{1}{3}(3)(1800)\left(\frac{3}{4}\right)$$

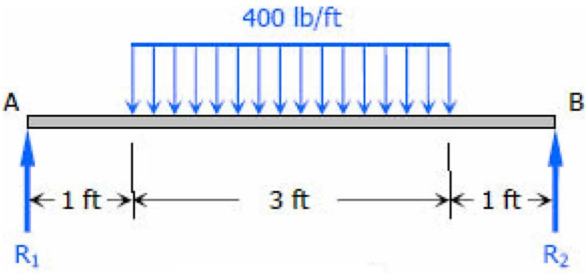
$$(Area_{AB})\bar{X}_A = 5450 \ \text{N.m}^3 \qquad Ans.$$

$$(Area_{AB})\bar{X}_B = \frac{1}{2}(4)(2800)\left(\frac{8}{3}\right) - \frac{1}{2}(2)(1000)\left(\frac{4}{3}\right) - \frac{1}{3}(3)(1800)\left(\frac{9}{4}+1\right)$$

$$(Area_{AB})\bar{X}_B = 7750 \ \text{N.m}^3 \qquad Ans.$$

Example 3

For the eam loaded as shown in the figure, compute the moment of area of the ***M*** diagrams between the reactions about both the left and the right reaction.



Solution

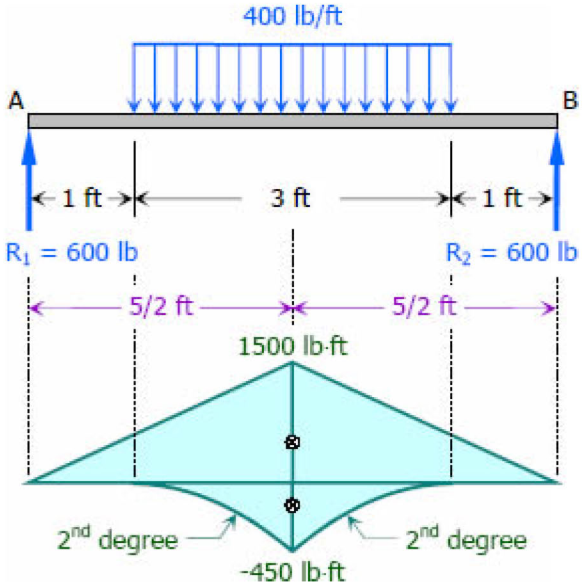
By symmetry

$$R_1 = R_2 = \frac{1}{2}(400)(3)$$

$$R_1 = R_2 = 600 \text{ lb}$$

and

$$(Area_{AB})\bar{X}_A = (Area_{AB})\bar{X}_B$$



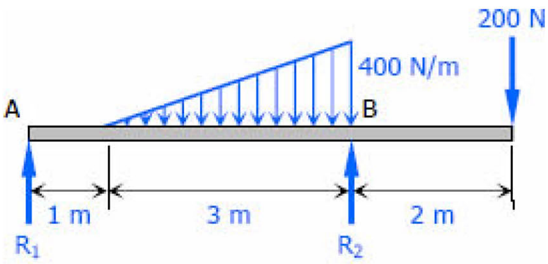
$$(Area_{AB})\bar{X}_A = \frac{1}{2}(5)(1500)\left(\frac{5}{2}\right) - \frac{1}{3}(3)(450)\left(\frac{5}{2}\right)$$

$$(Area_{AB})\bar{X}_A = 8250 \quad \text{lb.ft}^3 \quad \text{Ans.}$$

Thus,

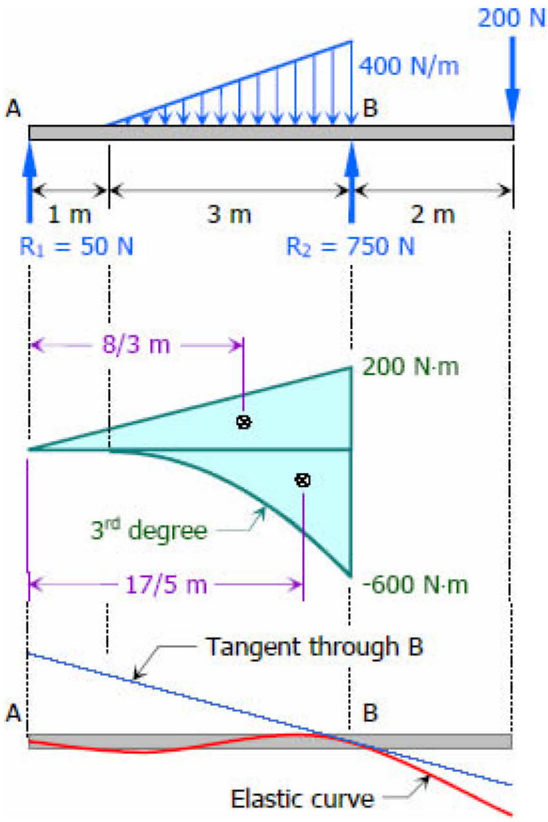
$$(Area_{AB})\bar{X}_B = 8250 \quad \text{lb.ft}^3 \quad \text{Ans.}$$

Example 4



For the beam loaded as shown in the figure, compute the value of $(\text{Area}_{AB})(\bar{X})_A$. From this result, is the tangent drawn to the elastic curve at ***B*** directed up or down to the right?

Solution



$$\Sigma M_{R2} = 0$$

$$4R_1 + 200(2) = \frac{1}{2}(3)(400)(1) \quad \Rightarrow R_1 = 50\ \text{N}$$

$$\Sigma M_{R1} = 0$$

$$4R_2 = 200(6) + \frac{1}{2}(3)(400)(3) \quad \Rightarrow R_2 = 750\ \text{N}$$

$$(Area_{AB})\bar{X}_A = \frac{1}{2}(4)(200)\left(\frac{8}{3}\right) - \frac{1}{4}(3)(600)\left(\frac{17}{5}\right)$$

$$(Area_{AB})\bar{X}_A = -463.33 \quad \text{N.m}^3 \qquad \text{Ans.}$$

The value of $(Area_{AB})(\bar{X})_A$ is negative; therefore point ***A*** is below the tangent through ***B***, thus **the tangent through B slopes downward to the right**.